

FIRST SEMESTER EXAMINATION, 2008-2009

ENGINEERING PHYSICS – I

Time : 2 Hours

Total Marks : 50

Note. This question paper contains 3 sections.

SECTION — A

Q.1. Attempt ALL parts. All parts carry equal marks: (10×1=10)

- (a) The volume of a cube, the proper length of each edge of which is l_0 when it is moving with a velocity v along one of its edges can be given as _____.

Ans. Volume, $V = l_0^3 \sqrt{1 - \frac{v^2}{c^2}}$

- (b) A particle of rest mass m_0 moves with speed $\frac{c}{\sqrt{2}}$. Its mass can be given _____

Ans. $m = \sqrt{2} m_0 = 1.44 m_0$

Pick the correct choice from following:

- (c) In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and screen is doubled. The fringe width is:
- (i) Unchanged (ii) Halved
(iii) Doubled (iv) Quadrupled

Ans. (iv) Quadrupled

- (d) Ray optics is valid when characteristic dimensions are:
- (i) Much smaller than the wavelength of light
(ii) Much larger than the wavelength of light

- (iii) Of the same order as the wavelength of light

- (iv) Of the order of 1 millimetre

Ans. (ii) Much larger than the wavelength of light.

- (e) The specific rotation of an optically active substance of length l_{cm} and concentration c with an optical rotation θ is:

(i) $\frac{l_c}{10\theta}$

(ii) $\frac{\theta l_c}{10}$

(iii) $\frac{10\theta}{l_c}$

(iv) $\frac{10}{\theta l_c}$

Ans. (iii) $\frac{10\theta}{l_c}$

- (f) In He-Ne laser, the helium and neon are in the ratio

(i) 1 : 7

(ii) 7 : 1

(iii) 1 : 10

(iv) 10 : 1

Ans. (ii) 7 : 1 or (iv) 10 : 1

- (g) The directionality of a laser beam is measured by

- (i) Visibility of interference fringes

- (ii) The size and aperture of the laser source

- (iii) The divergence angle of the beam with the distance from the source

- (iv) Nature of the lasing medium

Ans. (iii) The divergence angle of the beam with the distance from the source.

(h) Optical filters are made of

- (i) Metallic conductor
- (ii) Plastics doped with metallic impurities
- (iii) Dielectric material
- (iv) Magnetic oxides

Ans. (iii) Dielectric material

(i) Angle of acceptance is maximum for a filter of

- (i) The critical angle is minimum
- (ii) The critical angle is maximum
- (iii) The critical angle is zero
- (iv) The critical angle is negative

Ans. (i) The critical angle is minimum

(j) When a polaroid is rotated, the intensity of light is not formed to vary.

The incident light may be

- (i) Completely plane polarized
- (ii) Partially plane polarized
- (iii) Unpolarized
- (iv) None of these.

Ans. (iii) Unpolarized

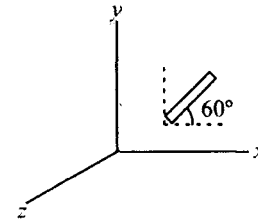
SECTION — B

Q.1. Attempt any THREE parts. All parts carry equal marks: (5×3=15)

(a) Calculate the percentage contraction of a rod moving with a velocity of 0.8c in a direction inclined at 60° to its own length.

Ans. Let L_o be the length of the rod in the frame S in which it is at rest. Let S' is the frame of reference which is moving with a speed of 0.8c in a direction making an angle 60° with X-axis. The components of length (L) along and perpendicular to the direction of motion are

$$L_o \cos 60^\circ \quad \text{and} \quad L_o \sin 60^\circ \quad \text{respectively.}$$



The apparent length of rod along the direction of motion

$$\begin{aligned} &= L_o \cos 60^\circ \sqrt{1 - \frac{v^2}{c^2}} \\ &= L_o \times \frac{1}{2} \sqrt{1 - (0.8)^2} \quad (\because v = 0.8c) \\ &= \frac{L_o}{2} \times 0.6 = 0.3L_o \end{aligned}$$

The apparent length in a direction perpendicular to the direction of motion

$$\begin{aligned} &= L_o \sin 60^\circ \\ &= \frac{\sqrt{3}}{2} L_o \end{aligned}$$

So in moving frame apparent length

$$L = \sqrt{(0.3L_o)^2 + \left(\frac{\sqrt{3}}{2} L_o\right)^2}$$

$$L = 0.916L_o$$

Percentage contraction in length

$$\begin{aligned} &= \frac{L_o - L}{L_o} \times 100 \\ &= \frac{L_o - 0.916L_o}{L_o} \times 100 = 8.4\% \end{aligned}$$

(b) A thin film of a soap solution is illuminated by white light at an angle

of incidence, $i = \sin^{-1}\left(\frac{4}{5}\right)$. In reflected

light, two dark consecutive overlapping

fringes are observed corresponding to wavelength $6.1 \times 10^{-7} \text{ m}$. The refractive index for soap solution is $4/3$. Calculate the thickness of the film.

Ans. We know that the condition for dark band is

$$2\mu t \cos r = n\lambda \quad \dots(1)$$

If n and $(n+1)$ are the orders of consecutive dark bands for wavelengths λ_1 and λ_2 respectively, then

$$2\mu t \cos r = n\lambda_1 \quad \dots(2)$$

$$\text{and } 2\mu t \cos r = (n+1)\lambda_2 \quad \dots(3)$$

Comparing equations (2) and (3), we get

$$n\lambda_1 = (n+1)\lambda_2$$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2} \quad \dots(4)$$

from equations (3) and (4), we have

$$2\mu t \cos r = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$t = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \cdot \frac{1}{2\mu \cos r} \quad \dots(5)$$

$$\text{But } \cos r = \sqrt{1 - \sin^2 r}$$

$$\cos r = \sqrt{1 - \frac{\sin^2 i}{\mu^2}} \quad \left(\because \mu = \frac{\sin i}{\sin r} \right)$$

$$\text{Given } \mu = \frac{4}{3} \text{ and } \sin i = \frac{4}{5}$$

$$\therefore \cos r = \sqrt{1 - \frac{(4/5)^2}{(4/3)^2}}$$

$$\cos r = \frac{4}{5}$$

\therefore Now from equation (5), we get

$$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 \times 10^{-5} - 6.0 \times 10^{-5}) \times 2 \times \frac{4}{3} \times \frac{4}{5}}$$

$$t = 0.0017 \text{ cm}$$

$$t = 1.7 \times 10^{-3} \text{ cm}$$

- (c) A light of wavelength 6000 \AA falls normally on a straight slit of width 0.10 mm . Calculate the total angular width of the central maximum and also the linear width as observed on a screen placed 1 metre away.

$$\text{Ans. } \lambda = 6000 \text{ \AA}$$

$$\lambda = 6 \times 10^{-5} \text{ cm}$$

$$\text{Slit width } (a) = 0.10 \text{ mm}$$

$$\text{Screen at distance } (D) = 1 \text{ metre}$$

Total angular width of the central maximum = ?

Linear width = ?

Condition of minima due to single slit is:

$$a \sin \theta = n\lambda$$

If θ is small $\sin \theta \simeq \theta$

for $n = 1$

$$a \sin \theta = n\lambda$$

$$a \cdot \theta = 1 \cdot \lambda$$

$$\theta = \frac{\lambda}{a}$$

$$= \frac{6 \times 10^{-5}}{0.01} = 6 \times 10^{-3} \text{ radian}$$

Total angular width of central maximum

$$2\theta = 1.2 \times 10^{-2} \text{ radian}$$

$$\text{or } 2\theta = 0.688 \text{ degrees}$$

$$\text{Linear half width} = \theta \cdot D$$

$$= 6 \times 10^{-3} \times 100$$

$$= 0.6 \text{ cm}$$

$$\text{Total linear width} = 2\theta \cdot D = 1.2 \text{ cm}$$

- (d) The indices of refraction of quartz for right handed and left handed circularly polarized lights of wavelength 6500 \AA travelling in the direction of optic axis have the following value $\mu_r = 1.53914$ and

$\mu_L = 1.53920$. Calculate the rotation of the plane of polarization of light in degrees produced by a plate 0.2 mm thick.

Ans. Given, $\lambda = 6500 \text{ \AA} = 6.5 \times 10^{-7} \text{ meter}$

$$\mu_L = 1.53920$$

$$\mu_R = 1.53914$$

Thickness $d = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ meter}$

In case of right-handed substances, the rotation of the plane of polarisation is given by relation:

$$\theta_R = \frac{\pi}{\lambda} (\mu_L - \mu_R) d$$

$$= \frac{3.14 \times (1.53920 - 1.53914) \times 0.2 \times 10^{-3}}{6.5 \times 10^{-7}}$$

$$\theta_R = 0.05796 \text{ radians}$$

or $\theta_R = 3.32 \text{ degree}$

- (e) A silica glass optical filter has a core refractive index of 1.500 and cladding refractive index of 1.450. Calculate the numerical aperture of the filter.

Ans. Given. Refractive index of core (n_1) = 1.6

Refractive index of cladding (n_2) = 1.450

Numerical aperture (N.A.) = ?

$$\text{N.A.} = \sqrt{n_1^2 - n_2^2}$$

$$\text{N.A.} = \sqrt{(1.5)^2 - (1.45)^2}$$

$$\text{N.A.} = \sqrt{0.1475} = 0.384$$

SECTION — C

Note: Attempt ALL questions. All questions carry equal marks. (5×5=25)

Q.3. Attempt any one part of the following:

- (a) Show from Lorentz transformation

that two events simultaneous ($t_1 = t_2$)

at different positions ($x_1 \neq x_2$) in a reference frame S are not, in general simultaneous in another reference frame S' .

Ans. Ans. From Lorentz transformation we have

$$t' = t - \frac{vx}{c^2 \sqrt{1 - v^2/c^2}}$$

So the transformation of time t_1 and t_2 in moving frame of reference is given as

$$t'_1 = \frac{t_1 - \frac{vx_1}{c^2}}{\sqrt{1 - v^2/c^2}}$$

and $t'_2 = \frac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1 - v^2/c^2}}$

$$\therefore t'_2 - t'_1 = \frac{(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

If $t_1 = t_2$. But $x_2 \neq x_1$

Then $t'_2 \neq t'_1$

\Rightarrow The two events are simultaneous in one frame of reference are not in several simultaneous in another frame.

- (b) What was the objective of conducting the Michelson Morley experiment? Describe the experiment. How is the negative result of the experiment interpreted?

Ans. Michelson Morley Experiment: Michelson Morley performed a historic optical experiment to check if the velocity of light measured with respect to earth moving through ether actually depended upon the velocity of the earth.

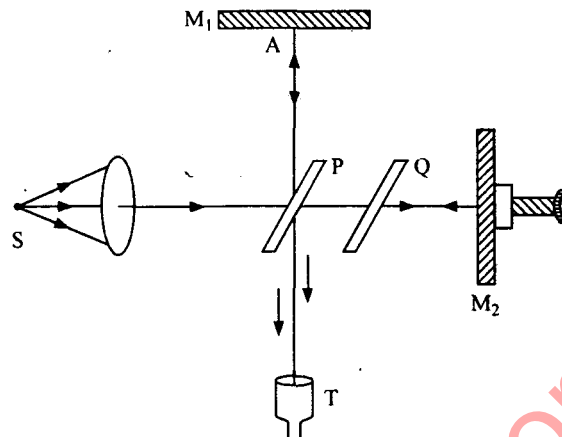


Fig. (1)

Experimental arrangement (Fig. 1) for the Michelson Morley experiment is shown below Fig. 1. S is any monochromatic source, a parallel beam falls upon a glass plate P , which is inclined 45° from the horizontal and plate is semisilvered polish. About half the light gets reflected at the back surface along mirror M_1 , whereas other half gets refracted at the both surfaces to emerge parallel to the original beam along M_2 . It also consists of plate Q such that P and Q are of equal thickness and of the same material, mounted parallel to each other.

The reflected beam moves towards mirror M_1 and at A it falls normally, hence it is reflected back to P and other being transmitted from P enters the telescope T . Similarly transmitted beam moves towards mirror M_2 and falls normally at B after entering the plate Q . The beam reflected at P crosses the plate P twice while other beam in the absence of Q lies wholly in air. So the reflected beam at P has to travel an extra optical path $2(\mu - 1)t$, μ is reflective index of the P and t is its thickness. The function of the plate Q is only to equalise the optical paths traversed by the both the beams if mirrors M_1 and M_2 are at equal distances from P .

Let us suppose that the two mirrors be at equal distances from the plate P , i.e. $PA = PB = l$ (say).

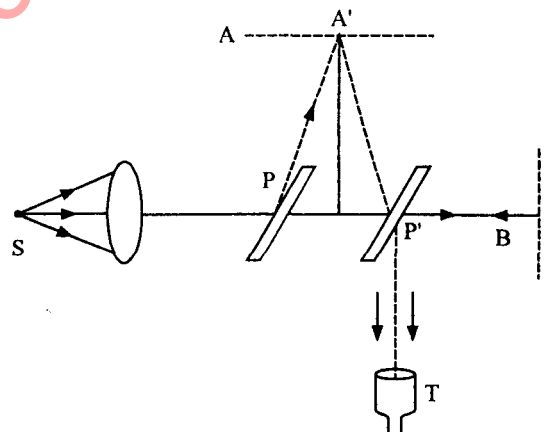


Fig. (2)

We have supposed that velocity of the apparatus is V . So the ray coming from P strikes the mirror M_1 at A' instead of A due to the motion of the earth. (Fig. 2)

Hence, total path travelled by reflected beam = $PA'P' = PA' + A'P' = 2 PA'$ (since $PA' = P'A'$)

From the triangle $PA'O$, we have, $(PA')^2 = (PO)^2 + (OA')^2$... (1)

Again supposing that time taken by the beam to reach from P to mirror M_1 , then

$$PA' = ct \text{ and } AA' = vt \text{ and } OA' = l$$

Therefore equation (1) can yields the form

$$c^2 t^2 = v^2 t^2 + l^2$$

or
$$t = \frac{l}{\sqrt{c^2 - v^2}}$$

If t_1 time is taken by the reflected beam to complete total distance, then

$$\begin{aligned} t_1 &= 2t = \frac{2l}{\sqrt{c^2 - v^2}} \\ &= \frac{2l}{c \sqrt{1 - \frac{v^2}{c^2}}} = \frac{2l}{c} \left\{ 1 - \frac{v^2}{c^2} \right\}^{-1/2} = \frac{2l}{c} \left(1 + \frac{v^2}{c^2} \right) \text{ (By binomial theorem)} \end{aligned} \quad \dots (2)$$

The transmitted beam has velocity $(c - v)$ because the direction of travel of light is along the direction of motion of apparatus, similarly $(c + v)$ along opposite direction of the light.

So total time taken by transmitted beam to travel the total path i.e., from P to B' and from B' to P' is t_2 ,

Therefore
$$\begin{aligned} t_2 &= \frac{l}{c - v} + \frac{l}{c + v} \text{ (since } PB = P'B' = l) \\ &= \frac{2l}{c^2 - v^2} = \frac{2l}{c^2 \left(1 - \frac{v^2}{c^2} \right)} = \frac{2l}{c^2} \left(1 - \frac{v^2}{c^2} \right)^{-1} \\ &= \frac{2l}{c} \left(1 + \frac{v^2}{c^2} \right) \end{aligned} \quad \dots (3)$$

Therefore, the time difference

$$\begin{aligned} \Delta t &= t_2 - t_1 = \frac{2l}{c} \left\{ 1 + \frac{v^2}{c^2} \right\} - \frac{2l}{c} \left\{ 1 + \frac{v^2}{2c^2} \right\} \\ &= \frac{2l}{c} \cdot \frac{v^2}{2c^2} = \frac{l}{c} \cdot \frac{v^2}{c^2} = \frac{lv^2}{c^3} \end{aligned} \quad \dots (4)$$

Due to this time difference, the waves which are in phase at the time separate at P will not be in phase on return. The time difference should appear as fringe shift from the position the fringe would occupy, if there were no other drift. So the difference of path will be in opposite direction, the total shift of fringe is equivalent to $\frac{2l v^2}{c^3 \lambda}$.

Negative results of Michelson Morley experiment:

The experiments were repeated at various locations and at different times of the year, but always yielded null results. The following interpretations were offered to explain the null results of the experiment.

1. Ether-Drage Hypothesis: The earth might drag a little ether along with it near its surface. In another words, the moving earth drags the ether along with it and hence there is no relative motion between the earth and ether.

2. Lorentz-Fitzgerald Contraction Hypothesis: A brilliant suggestion to explain the negative results of Michelson Morley experiment, was given by Fitzgerald and Lorentz. They proposed that the dimensions of all material bodies get contracted by a factor

$\sqrt{1 - \frac{v^2}{c^2}}$ in a direction parallel to the relative velocity. In Michelson Morley experiment, the Fitzgerald contraction means that when the apparatus is in motion, the distance travelled

by the beam B is not l , but $l\sqrt{1 - \frac{v^2}{c^2}}$. Thus

Fitzgerald contraction theory is able to explain the null results of Michelson Morley experiment, it is purely hypothetical.

3. Constancy of Speed of Light: As Einstein proposed that light's speed is invariant, means speed of light in never depends or changes upon the motion of the source.

The essential conclusion, which follows from the results of Michelson Morley experiment is that it is not possible to measure the relative velocity of the earth relative to ether.

Q.4. Attempt any one part of the following:

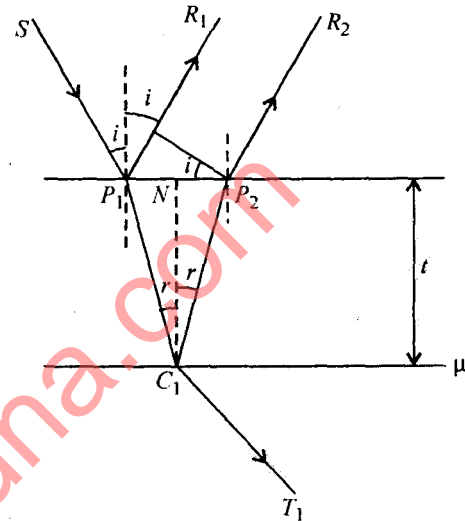
- (a) Discuss the phenomena of interference of light due to thin films of uniform thickness in reflected light and find the conditions of maxima and minima.

Ans. Interference in Thin Films:

(i) Interference in Reflected rays:

∴ Path difference between the reflected rays P_1R_1 and $P_1C_1P_2R_2$ is given by

$$\Delta' = \mu(P_1C_1 + C_1P_2) - P_1A \quad \dots(1)$$



In $\Delta P_1C_1P_2$

$$\angle P_1C_1N = \angle NC_1P_2 = r$$

$$P_1C_1 = C_1P_2 = \frac{t}{\cos r} \quad \dots(2)$$

and $\tan r = \frac{P_1N}{t} = \frac{P_2N}{t}$

$$\therefore P_1N = P_2N = t \tan r \quad \dots(3)$$

and $\angle P_1P_2A = i$

In ΔAP_1P_2

$$\sin i = \frac{P_1A}{P_1P_2}$$

$$\begin{aligned} P_1A &= P_1P_2 \sin i \\ &= (P_1N + NP_2) \sin i \\ &= 2t \tan r \sin i \end{aligned} \quad \dots(4)$$

Substituting eqs. (2) and (4) in eq. (1), we get

$$\Delta' = \mu \frac{2t}{\cos r} - 2t \tan r \cdot \sin i$$

$$= \frac{2\mu t}{\cos r} \left[1 - \frac{\tan r \sin i \cos r}{\mu} \right]$$

$$= \frac{2\mu t}{\cos r} \left[1 - \frac{\frac{\sin r}{\cos r} \sin i \cos r}{\sin i} \right]$$

$$\left(\because \mu = \frac{\sin i}{\sin r} \right)$$

$$\Delta' = \frac{2\mu t}{\cos r} [1 - \sin^2 r]$$

$$\Delta' = 2\mu t \cos r \quad \dots(5)$$

According to Stoke's Theorem, when a light suffers reflection at a denser medium i.e. at P_1 a phase change of π or path difference $\lambda/2$ takes place. Hence

Total path difference

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

(i) Constructive interference: When $\Delta = n\lambda$, a constructive interference is produced and entire film will appear bright.

$$\text{Thus } \Delta = 2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

(For constructive interference)

$$2\mu t \cos r = n\lambda \pm \frac{\lambda}{2}$$

$$2\mu t \cos r = \left(n \pm \frac{1}{2} \right) \lambda$$

$$\text{If } 2\mu t \cos r = \left(n + \frac{1}{2} \right) \lambda \quad (n = 0, 1, 2, 3, \dots)$$

$$\text{and if } 2\mu t \cos r = \left(n - \frac{1}{2} \right) \lambda \quad (n = 1, 2, 3, \dots)$$

(ii) Destructive Interference: When

$$\Delta = \left(n \pm \frac{1}{2} \right) \lambda$$

a destructive interference is produced and entire film appears dark.

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2} = \left(n \pm \frac{1}{2}\right)\lambda$$

$$2\mu t \cos r = n\lambda \quad (n = 0, 1, 2, 3, \dots)$$

- (b) What do you understand by missing order spectrum? What particular spectra would be absent if the width of the transparencies and opacities of the grating are equal. Show that only first order spectra is possible if the width of the grating element is more than wavelength of light and less than twice the wavelength of light.

Ans. Missing Order Spectrum: If the angle of diffraction is such that the minima due to diffraction component in the intensity distribution fall at the same positions of principal maxima due to interference component, then that order of principal maxima will be absent.

Thus for absent spectrum of n th order

$$a \sin \theta = m\lambda \quad \dots(1)$$

$$(a + b) \sin \theta = n\lambda \quad \dots(2)$$

From equations (1) and (2), we have

$$\frac{a}{a + b} = \frac{m}{n}$$

If $a = b$, then

$$n = 2m$$

But $m = 1, 2, 3, \dots$

then $n = 2, 4, 6, \dots$

i.e. 2nd, 4th and 6th order of principal maxima will be absent.

Condition for maximum number of order is given by

$$n_{\max} = \frac{a + b}{\lambda} \quad (\sin \theta = 1)$$

Given if

$$a + b < 2\lambda$$

or $\frac{a + b}{2\lambda} < 1$

$$\frac{a + b}{\lambda} < 2$$

$$\therefore n_{\max} < 2$$

i.e., only first order is possible.

Q.5. Attempt any one part of the following:

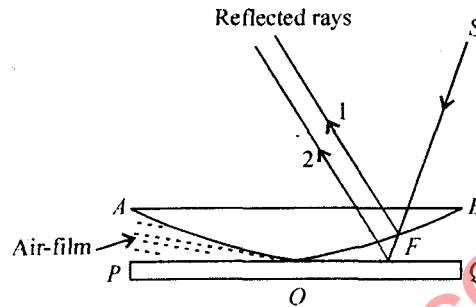
- (a) Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.

Ans. Newton rings are formed due to the interference of light rays reflected from the upper and lower surfaces of the air-flow formed between the convex surface of a plano-convex lens and glass

plate.

The incident ray SF is divided into two coherent rays 1 and 2 by reflection from the upper and lower surfaces of the wedge-shaped air film.

These reflected rays 1 and 2 interfere and produces bright and dark circular rings around the point of contact.



The effective path difference between the interfering rays in reflected light is

$$\Delta = 2\mu t \cos(r + \theta) \pm \frac{\lambda}{2}$$

where $\mu \rightarrow$ refractive index of film

$t \rightarrow$ thickness of film

$r \rightarrow$ angle of reflection

$\theta \rightarrow$ angle of wedge

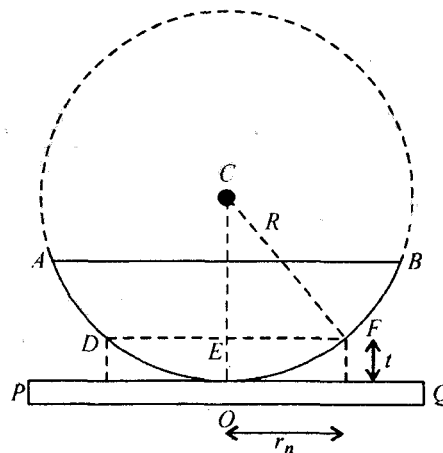
For normal incidence ($i = r = 0^\circ$), and for small angle of wedge (θ), i.e., $\theta \simeq 0$

$$\cos(r + \theta) = \cos \theta \simeq 1$$

So, effective path difference

$$\Delta = 2\mu t \pm \frac{\lambda}{2}$$

Diameter of Dark Rings: Let R be the radius of curved surface AOB of the lens and t the thickness of the film at any point F .



$$CF = R = CO$$

$$CE = CO - EO$$

$$CE = R - t$$

In $\triangle CEF$,

$$R^2 = r_n^2 + (R - t)^2$$

$$R^2 = r_n^2 + R^2 + t^2 - 2Rt$$

$$r_n^2 = 2Rt - t^2$$

$$r_n^2 = 2Rt \quad \dots(1) \quad (R \gg t \text{ so } t^2 \text{ can be neglected})$$

But for dark rings

$$\text{Path diff. } (\Delta) = (2n \pm 1) \frac{\lambda}{2} \quad \dots(2)$$

$$\text{But } \Delta = 2\mu t \pm \frac{\lambda}{2} \quad \dots(3)$$

From eqs. (2) and (3), we get

$$2\mu t \pm \frac{\lambda}{2} = (2n \pm 1) \frac{\lambda}{2}$$

$$2\mu t = n\lambda$$

$$2\mu \cdot \frac{r_n^2}{2R} = n\lambda \quad (r_n^2 = 2Rt)$$

$$r_n^2 = \frac{n\lambda R}{\mu}$$

If diameter of n th dark ring is D_n , then

$$D_n = 2r_n$$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}}$$

For air-film, $\mu = 1$

$$D_n = \sqrt{4n\lambda R}$$

$$\therefore D_n \propto \sqrt{n}$$

where $n = 1, 2, 3 \dots$

$$D_n \propto 1, \sqrt{2}, \sqrt{3} \dots$$

\therefore Thus the diameters of dark rings are proportional to the square root of the natural numbers.

(b) Discuss the phenomena of Fraunhofer diffraction at a single slit and show that the relative intensities of successive maxima are nearly

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \dots$$

Ans. The fringe-width is given by

$$\omega = \frac{D\lambda}{2d} \quad \text{or} \quad \lambda = \frac{\omega \cdot 2d}{D}$$

$$\text{Given } D = 170 \text{ cm}$$

$$2d = 0.04 \text{ cm}$$

$$\omega = 0.25 \text{ cm}$$

$$\therefore \lambda = \frac{0.25 \times 0.04}{170} = 5880 \text{ \AA}$$

Condition for missing order is

$$\frac{e + d}{e} = \frac{n}{m}$$

$$\text{Given } d = 0.4 \text{ mm and } e = 0.08 \text{ mm}$$

$$\therefore d = 5e$$

Substituting equation (1), we get

$$\frac{e + 5e}{e} = \frac{n}{m} \quad \text{or} \quad n = 6m$$

$$m = 1, 2, 3, \dots$$

$$\text{or } n = 6, 12, 18, \dots \text{ Ans.}$$

6. Attempt any one part of the following:

(a) Define specific rotation. Describe the construction and working of Laurent's half-shade polarimeter.

Ans. Specific Rotation: "The specific rotation of an optically active substance at a given temperature for a given wavelength of light is defined as the rotation in degree produced by a path of *one decimeter length* in a substance of *unit density*."

If ' θ ' is the rotation produced by ' l ' decimeter length of an optically active substance, the

concentration of its solution is 'C' gm/cc, then specific rotation at a given temperature for a given wavelength is expressed as

$$S = \frac{\theta}{l_c} \text{ degree (decimeter)}^{-1} (\text{gm/cc})^{-1}$$

If l is in cm, then

$$S = \frac{10\theta}{l_c}$$

(i) **Laurent's half shade polarimeter:** The experimental arrangement of this polarimeter is same as shown in figure (4), but to increase its sensitivity a particular device (half shade device) is placed in between the polariser and the polarimeter tube T .

Half Shade Device: It consists of two semi-circular plates of same radius cemented together along the diameter so as to form a complete circular plate. Fig. (5)

One semi-circular plate is a *half wave plate* made from "quartz" in which optic axis is parallel to its face and the other semi-circular plate is a *glass plate*. The half wave plate introduces a *phase difference* of π or a *path difference* of $\lambda/2$ between the ordinary and extraordinary components. Thickness of the glass plate is so adjusted that it should absorb same amount of light as that by quartz plate.

In this way the intensity of transmitted light from these semicircular plates remain same.

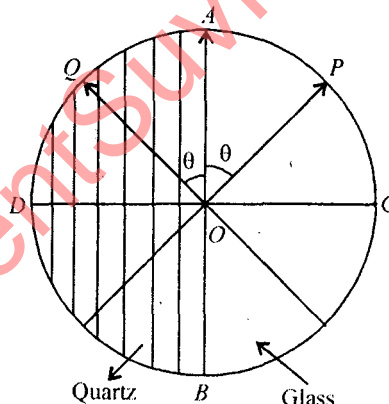


Fig. (5)

Circular plate made by this device is called *Laurent's half shade device*.

Laurent's plate is kept in between the polariser and the tube in such a way that plane of vibrations of incident plane polarised light remain perpendicular to the face of Laurent's plate.

Working: Suppose the plane of vibrations of transmitted plane polarised light from polarizer incident normally on the plate is in the direction OP making an angle θ with the diameter AOB . This plane of vibration is transmitted in the same direction OP without any rotation by the semi-circular glass plate whereas the semicircular quartz plate splits the incident plane polarised light into ordinary and extraordinary components. These vibrations are transmitted through the plate with a phase difference of π and the resultant vibration is transmitted in the direction OQ . In this way the plane of vibration transmitted through the two semicircular plates makes equal angle with AOB , i.e.,

$$\angle AOP = \angle AOQ = \theta$$

Clearly two plane polarised rays are obtained by transmission through Laurent's plate.

- (i) one transmitted through glass whose plane of vibration is OP and
- (ii) another transmitted through quartz plate whose plane of vibration is OQ . These vibrations are studied by analyser.

If the principal section A_1OA_2 of the analyser A is perpendicular to AOB as shown in fig. 6, then both half portions of the field of view will be seen equally bright because the components OE_1 and OE_2 of the vibrations OP and OQ , respectively, along A_1OA_2 are equal.

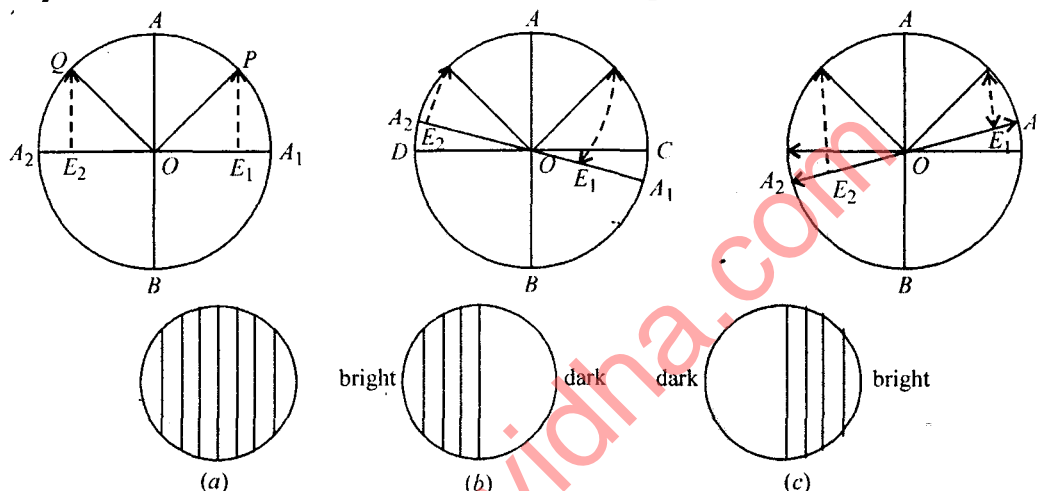


Fig. (6)

If the principal section A_1OA_2 of the analyser A is slightly rotated in the clockwise direction as shown in fig. 6(b), then component OE_2 increases and OE_1 decreases. As a result the left half of the field of view appears brighter than the right half.

On the other hand, if the principal section of the analyser A is slightly rotated in the anti-clockwise direction as shown in fig. 6(c), then the component OE_2 becomes less than the component OE_1 and right half of the field of view now appears brighter than the left half.

To determine the specific rotation of a solution: To find the specific rotation of an optically active solution of known concentration (ray sugar solution), the optical rotation ' θ ' is found and using equation (below) the desired quantity may be found out.

$$S = \frac{\theta}{l \times c} \frac{\text{degree}}{\text{decimeter} \times (\text{gm/cm}^3)}$$

The polariser tube is filled with plain water and the analyser is set in the position of equal brightness of the field of view and the reading of circular scale with the help of vernier is noted down.

Now polarimeter tube is filled with given solution and the analyser is again set in the position for equal brightness of the field of view and the reading of circular scale with the help of vernier scale is noted down.

The difference between these two readings give the optical rotation ' θ ' produced by the given length l (in decimeter) of solution. Now specific rotation S of the solution for a particular wavelength and temperature can be found as

$$S = \frac{\theta}{l_c}$$

Disadvantage: It can be used for one particular wavelength for which the quartz half plate serves as a half-wave plate.

(b) Explain the principle of optical pumping and stimulated emission of radiation. Discuss the properties of laser radiation and mention some of its applications.

Ans. Optical Pumping: Under normal conditions of thermal equilibrium, the number of atoms in higher energy state is smaller than the number of atoms in lower energy states, i.e.

$$\begin{array}{ccc} E_m < E_n & N_n & m_1 E_n \\ \therefore N_n < N_m & N_m & m_1 E_m \end{array}$$

“The situation in which the number of atoms in higher energy state exceeds that in the lower energy state is known as population inversion.”

“The process by which atoms are brought from lower energy states to higher energy states is called *optical pumping*.”

There are various types of pumping process but most suitable and commonly used is *optical pumping*.

Stimulated Emission: In 1917, Einstein was the first to point out a third possibility, i.e. induced emission, in which an incident photon of energy $h\nu$ causes a transition from upper state E_2 to lower state E_1 as shown in figure.

In this process, the radiated light waves are exactly in phase with incident ones. As a result these radiations passing through an assembly of atoms is amplified.

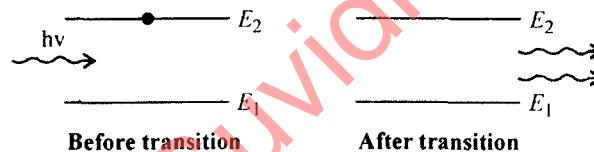


Fig. Stimulated Emission

Properties of LASER: The laser beam has the following main characteristics:

1. High monochromaticity
2. High intensity
3. High degree of coherence
4. High directionality
5. A laser beam is very narrow and can travel to long distance without any spread.

Applications of LASER: Because of high temporal coherence and large spatial coherence, laser beams have wide applications in various branches of science, spectroscopy, engineering and computer.

1. Industrial application: For drilling, extremely fine holes in diamonds, teeth, paper clips, hard sheets.

2. Medical applications: For micro surgery, for treatment of kidney stone, tumour etc.

3. In communication: For micro-wave communication, earth satellites, distance between earth and moon can be measured with laser.

4. Chemical application

5. In spectroscopy

7. Attempt any one part of the following:

(a) Explain basic principle of optical fiber. Discuss fiber classification.

Ans. Basic Principle of Optical Fiber: The optical fiber has two concentric layers called the core and the cladding. The inner core is the light-carrying part. The surrounding cladding provides the difference in refractive index that allows *total internal reflection* of light through the core.

The refractive index of the cladding is less than 1% lower than that of the core.

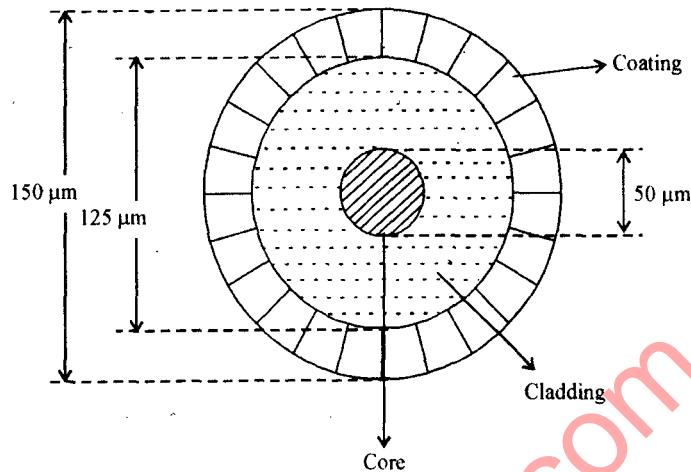


Fig. Cross-sectional view of an optical fiber (Typical core and cladding diameters).

Classification of optical fiber: There are two ways to classify optical fiber:

1. According to their material makeup
2. Refractive index of core and cladding

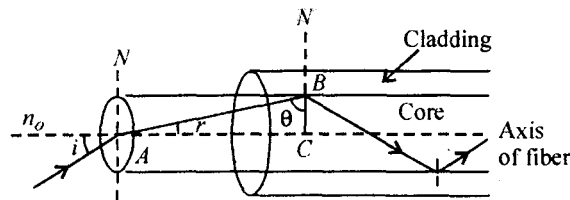
1. According to their material makeup:

- (a) **Glass-fiber:** Glass core and glass cladding.
- (b) **Plastic-Clad Silica (PCS):** Glass core and plastic cladding
- (c) **Plastic fiber:** Plastic core and plastic cladding.

2. According to refractive index: The second way to classify fibers is by the refractive index of core and cladding.

- (a) Step-index fiber
- (b) Graded-index fiber
- (c) Single mode step index fiber

Propagation of light through optical fiber (Numerical Aperture (N.A)): Propagation of light waves through the fiber is based on the total internal reflection (TIR). The main purpose of an optical fiber is to accept and transmit light from the source.



The light gathering ability of a fiber depends on two factors:

1. Core size
2. Numerical aperture (N.A.)

Let us consider a ray incident on the aperture of one end of the fiber making an angle i with the axis of the fiber. The ray refracts at an angle ' r ' and strikes the core-cladding interface at an angle θ . If ' θ ' is greater than critical angle θ_c , then the ray undergoes total internal reflection.

Suppose, R.I. of Core $\rightarrow n_{\text{core}}$ (R.I \rightarrow refractive index)

R.I. of Cladding $\rightarrow n_{\text{clad}}$
 and $n_{\text{core}} > n_{\text{clad}}$
 Let n_o be R.I. of medium from which light is launched into the fiber.
 Applying Snell's law (on core and outside interface)

$$n_o \sin i = n_{\text{core}} \sin r$$

or
$$\frac{\sin i}{\sin r} = \frac{n_{\text{core}}}{n_o} \quad \dots(1)$$

Applying Snell's law on core-cladding interface

$$n_{\text{core}} \sin \theta = n_{\text{clad}} \sin r'$$

If $\theta = \theta_c$ (critical angle)
 Then $r' = 90^\circ$

$$\sin \theta_c = \frac{n_{\text{clad}}}{n_{\text{core}}} \quad \text{or} \quad \theta_c = \sin^{-1} \left(\frac{n_{\text{clad}}}{n_{\text{core}}} \right) \quad \dots(14)$$

But in $\triangle ABC$, $\sin r = \sin (90^\circ - \theta) = \cos \theta$

From eq. (1): $\sin i = \frac{n_{\text{core}}}{n_o} \sin r$

$$\sin i = \frac{n_{\text{core}}}{n_o} \cos \theta \quad \dots(3)$$

When $\theta = \theta_c$, $i = i_{\text{max}} = \alpha$ (Acceptance angle)

From (3), $\sin i_{\text{max}} = \frac{n_{\text{core}}}{n_o} \cos \theta_c$

$$= \frac{n_{\text{core}}}{n_o} \sqrt{1 - \sin^2 \theta_c} = \frac{n_{\text{core}}}{n_o} \sqrt{1 - \frac{n_{\text{clad}}^2}{n_{\text{core}}^2}} = \frac{n_{\text{core}}}{n_o} \sqrt{\frac{n_{\text{core}}^2 - n_{\text{clad}}^2}{n_{\text{core}}^2}}$$

$$\sin i_{\text{max}} = \sqrt{\frac{n_{\text{core}}^2 - n_{\text{clad}}^2}{n_o^2}}$$

If light is launched from air i.e. $n_o = 1$

$$\sin i_{\text{max}} = \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2}$$

$$i_{\text{max}} = \sin^{-1} (\sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2})$$

(Acceptance angle)

Acceptance angle: The maximum angle that a light ray can have relative to the axis of the fiber and propagate down the fiber. This angle is a measure of the light gathering.

(b) Explain acceptance angle and acceptance cone of a fiber. What do you mean by numerical aperture of a fiber. Derive expressions for them.

Physical constants:

Planck's constant $h = 6.63 \times 10^{-34}$ JS

Velocity of light in free space

$$c = 3 \times 10^8 \text{ m/s}$$

Rest mass of electron

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

Ans. Try yourself.